SPECTRAL MEAN ABSORPTION COEFFICIENTS FOR OPTICALLY THIN MEDIA

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Steady-state heat transfer is considered in a blackbody spectrum for local thermodynamic equilibrium in a nonscattering medium with an index of refraction of unity. The absorption coefficients of the medium are investigated for incident fluxes with two different spectra: a blackbody spectrum and one given by an element of volume.

Spectral mean absorption coefficients are of interest in the approximate description of radiative heat exchange in an actual medium by the gray-gas equations. The Planck mean absorption coefficient is used in the well-known approximation of an optically thin medium. This approximation can be made more precise and widely applicable by taking account of the dependence of the absorption coefficients on the path length and the difference in temperatures of the bodies of the system. Our previous paper [1] is supplemented in the following respects: 1) the coefficients are related to the energy transport equations; 2) an exact relation is found between the coefficients; 3) the estimates of the dependence of the coefficients on the path length are refined; 4) the estimates of the dependence of the coefficients on the temperature of the source and of the object being irradiated are refined.

In an isothermal system of bodies the energy transport equation has the form

$$d\varepsilon/dx = \alpha_c - \alpha'\varepsilon \tag{1}$$

$$\varepsilon = \frac{\pi I}{\sigma T^4}, \quad I = \int_0^\infty I_\omega d\omega, \quad \frac{\sigma T^4}{\pi} = \int_0^\infty I_{0\omega} d\omega$$

$$\alpha_c = \frac{\pi}{\sigma T^4} \int_0^\infty I_{0\omega}\alpha_\omega d\omega, \quad \alpha' = \frac{1}{I}\int_0^\infty \alpha_\omega I_\omega d\omega$$

Here $I_{0\omega}$ is the Planck function, I_ω is the spectral intensity in cm · W/m² · sr, ω is the wave number in cm^{-1}; for a single absorbing component $x = \int p\,dl$ in m · atm, p is the partial pressure, α_ω is the spectral absorption coefficient in (m · atm)$^{-1}$, and α_c is the Planck mean absorption coefficient. Contributions to the dimensionless flux ε are made by the boundary surface (ε_1) and the volume of the medium (ε_2). In accord with the equality $\varepsilon = \varepsilon_1 + \varepsilon_2$, Eq. (1) is separated into

$$\frac{d\varepsilon_1}{dx} = -\alpha\varepsilon_1, \qquad \frac{d\varepsilon_2}{dx} = \alpha_c - \alpha^*\varepsilon_2 \tag{2}$$

The solutions of equations (2) relate the fluxes ε_1 and ε_2 to the corresponding transmissibilities D and D* and the absorption coefficients of the medium: α for the flux with the initial blackbody spectrum, and α^* for the flux with the initial spectrum of an element of volume

$$\varepsilon_1 \equiv D = \exp\left(-\int_0^x \alpha(x_1)\,dx_1\right), \quad \varepsilon_2 = \int_0^x \alpha_c D^*(x, x_1)\,dx_1$$

$$D^*(x, x_1) = \exp\left(-\int_{x_1}^x \alpha^*(x_2)\,dx_2\right), \text{ or } \quad D^*(x) = \exp\left(-\int_0^x \alpha^*(x_1)\,dx_1\right)$$

Sverdlovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 149-151, May-June, 1971. Original article submitted October 13, 1970.

For an actual medium the coefficients α and α^* can differ by more than an order of magnitude. Therefore, the separation of Eq. (1) into two equations (2) is expedient.

The first problem is to estimate the dependence of α and α^* on the path length when it is small. As $x \to 0$,

$$\alpha \to \alpha_c, \quad \alpha^* \to \alpha_* = \frac{\pi}{\alpha_c \sigma T^4} \int_0^\infty I_{0\omega}\, \alpha_\omega^2 d\omega$$

The coefficients α_c and α_* are the maximum values of α and α^*. In the gray-gas approximation $\alpha_c = \alpha_*$. For CO_2 and H_2O the difference is very large, namely

$$\frac{\alpha_*}{\alpha_c} \approx 25 \text{ and } 16 \tag{3}$$

For an isothermal system of bodies the coefficients are related by

$$\int_0^x [\alpha(x_1) - \alpha^*(x_1)]\, dx_1 = \ln \frac{\alpha}{\alpha_c}$$

By using coefficients averaged over a range x

$$\langle\alpha\rangle = \frac{1}{x} \int_0^x \alpha(x_1)\, dx_1, \quad \langle\alpha^*\rangle = \frac{1}{x} \int_0^x \alpha^*(x_1)\, dx_1$$

the relations are simplified

$$D = e^{-\langle\alpha\rangle x}, \quad D^* = e^{-\langle\alpha^*\rangle x}$$
$$\langle\alpha\rangle - \langle\alpha^*\rangle = \frac{1}{x} \ln \frac{\alpha}{\alpha_c} \tag{4}$$

For thin enough media equations (4) are satisfied by the polynomials

$$\frac{\alpha}{\alpha_c} \approx 1 - (\alpha_* - \alpha_c)\, x + (\alpha_* - \alpha_c)^2 \frac{x^2}{2}$$
$$\frac{\langle\alpha\rangle}{\alpha_c} \approx 1 - (\alpha_* - \alpha_c) \frac{x}{2} + (\alpha_* - \alpha_c)^2 \frac{x^2}{6} \tag{5}$$
$$\frac{\alpha^*}{\alpha_*} \approx 1 - \frac{\alpha_c}{\alpha_*} (\alpha_* - \alpha_c)\, x, \quad \frac{\langle\alpha^*\rangle}{\alpha_*} \approx 1 - \frac{\alpha_c}{\alpha_*} (\alpha_* - \alpha_c) \frac{x}{2}$$

It was estimated in [1] that

$$\langle\alpha\rangle = \alpha_c (1 - \tfrac{1}{2}\alpha_* x), \quad \langle\alpha^*\rangle = \alpha_*$$

Taking account of (3) the refinement obtained here is small, but as the gray-gas condition is approached $\alpha_* \to \alpha_c$ and the refinement becomes very important. The limit of applicability of the estimates in terms of the value of x is described approximately by the inequality

$$(\alpha_* - \alpha_c)\, x \lesssim 0.5$$

It is clear that, as the gray-gas condition is approached, the "approximation of an optically thin medium" becomes valid for a medium of arbitrary thickness. Specifically the validity of the estimates is established by comparing the degree of blackness of the medium

$$\varepsilon = 1 - e^{-\langle\alpha\rangle x}$$

with the experimental values. For example, for water vapor, $\alpha_* = 10.8$ (m · atm)$^{-1}$ and $\alpha_c/\alpha_* = 0.6$ at 2000°C.

Estimate (5) gives good accuracy up to x = 0.1 m · atm. The usual approximation $\varepsilon = 1 - \exp(-\alpha_c x)$ under these same conditions is much more crude.

The coefficients α and α^* are given subscripts i and k corresponding to the temperature T_i of the radiation source and T_k of the object being irradiated. We consider below an estimate of the function $\alpha_{ik}(x, T_i, T_k)$ for a gas with a vibrational-rotational spectrum. As a starting point we take the approximate formula for the absorptivity of a gas

$$\alpha_{ik} = \xi^m \varepsilon (T_i, x'), \quad \xi = T_k/T_i, \quad x' = x\xi^u$$

Here x' is the equivalent isothermal path length; the exponents m and u are found either experimentally or theoretically [1]. We obtain the equation

$$\langle \alpha_{ik} \rangle (x) = \xi^u \langle \alpha \rangle (x') + \frac{1}{x} \ln \frac{\alpha_{ik}(x)}{\xi^{m+u} \alpha (x')} \tag{6}$$

Here

$$\alpha_{ik} = \frac{d(\langle \alpha_{ik} \rangle x)}{dx}, \quad \alpha(x') = \frac{d(\langle \alpha \rangle x')}{dx'} = \frac{d(\langle \alpha \rangle x)}{dx}$$

$$\langle \alpha_{ik} \rangle = \frac{1}{x} \int_0^x \alpha_{ik}(x_1)\, dx_1, \quad \langle \alpha \rangle (x') = \frac{1}{x'} \int_0^{x'} \alpha (x_1')\, dx_1'$$

If the medium is thin enough, Eq. (6) is satisfied approximately by

$$\alpha_{ik} = \xi^{m+u} \alpha_c [1 - x' (\alpha_* - \alpha_c \xi^m)],$$

$$\langle \alpha_{ik} \rangle = \xi^{m+u} \alpha_c [1 - {}^1/_2 x' (\alpha_* - \alpha_c \xi^m)]$$

The coefficients α_c and α_* are evaluated at temperature T_i. The coefficient $\langle \alpha_{ik}^* \rangle$ is discussed in [1].

LITERATURE CITED

1. S. P. Detkov, "Average absorption coefficient for optically thin media," Prikl. Mekhan. i Tekh. Fiz., No. 1 (1970).